Preference Completion: Large-scale Collaborative Ranking from Pairwise Comparisons

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The Problem

Given: for each user, a small number of pairwise comparisons:

"User i prefers item j_1 over j_2"

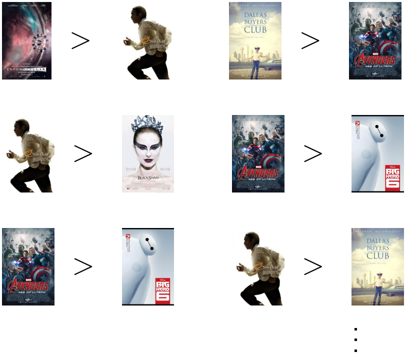
To find: personalized preference order for each user.

Alice (x_1^*)

Bob (x_2^*)

Charlie (x_3^*)

⋮



The Problem

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To find: personalized preference order for each user.

- Many kinds of user input can be turned into pairwise comparisons:
  - click/no-click: Each "click" is preferred to a (randomly chosen) "no-click"
  - one-of-many: chosen item preferred to others presented
  - numerical ratings: for each user, higher rated item preferred to lower rated one.
- Pairwise comparisons less subjective than numerical ratings

A Classic Model in Ranking

Bradley-Terry-Luce (BTL) model: for a single user setting

- Assume a ground-truth score vector $x^* \in \mathbb{R}^d$.
- Governs pairwise preferences:

$$Pr(j_1 \succ j_2) = \frac{1}{1 + \exp(-(x^*_{j_1} - x^*_{j_2}))}$$

- Popular for rank aggregation (fitting a single rank order to inconsistent preference data)

Our Approach

- Each user has its own, personal score vector $X_{i:}$. Items with higher score more likely to be preferred by that user.
- Taken together, the vectors form a **low-rank matrix**:
for d_1 users and d_2 items,

$$X \in \mathbb{R}^{d_1 \times d_2} \quad \text{and} \quad \mathrm{rank}(X) \leq r \qquad (r \ll d_1, d_2)$$

- Low-rank allows for generalization from a very small number of per-user comparisons (similar to the case of matrix completion)

Our contribution

- **Convex ERM**
  - We prove it has nearly optimal sample complexity: each user needs to make only $O(r \log^2(d_1 + d_2))$ pairwise comparisons
- **Alternating SVM (AltSVM)**
  - A scalable non-convex algorithm for the hinge loss, which we found works best in the practical large-scale settings.
  - Parallel implementation: near-linear speedup with number of cores in shared-memory machine
  - Outperforms existing (rating-based) algorithms both statistically and computationally.

Problem Setting

- d_1 users, d_2 items

Input

- $\Omega \subseteq [d_1] \times [d_2] \times [d_2]$: Set of (user, item 1, item 2) triples
- $\mathcal{Y} \triangleq \{Y_{ijk} \in \{+1, -1\} : (i, j, k) \in \Omega\}$: Pairwise comparisons

$$Y_{ijk} = \begin{cases} +1 & \text{“user } i \text{ prefers item } j \text{ to item } k\text{”} \\ -1 & \text{“user } i \text{ prefers item } k \text{ to item } j\text{”} \end{cases}$$

Output

- Predicted “score matrix” $X \in \mathbb{R}^{d_1 \times d_2}$

$$X_{ij} > X_{ik} \qquad \text{“user } i \text{ more likely to prefer item } j \text{ to item } k\text{”}$$

Convex ERM

$$\min_{X \in \mathbb{R}^{d_1 \times d_2}} \sum_{(i,j,k) \in \Omega} \mathcal{L}(Y_{ijk}(X_{ij} - X_{ik}))$$

$$\text{subject to} \quad \|X\|_* \leq \sqrt{\lambda d_1 d_2}$$

- Convex optimization over $d_1 \times d_2$ dimensional space
- Parameters: set $\lambda = O(r)$.
  Reason: If $\text{rank}(X) = r$ and $\|X\|_\infty \leq C$, then $\|X\|_* \leq C\sqrt{r d_1 d_2}$
- $\mathcal{L}$: appropriately chosen loss function.
  E.g. logistic for the BTL model. Our results for more general losses.

Convex ERM

Statistical performance: setup

- Each user-item-item triple (i, j, k) is sampled with probability p_{ijk}.
- No user-item pair is sampled too frequently.

$$\sum_k p_{ijk} \leq \kappa \frac{m}{d_1 d_2} \quad \text{(for fixed } m = \mathbb{E}|\Omega|\text{)}$$

- Expected risk: with Ω and Y's chosen as above, for any matrix X,

$$\mathbb{E}_{\Omega,\mathcal{Y}} R(X) \;:=\; \text{expected value of} \sum_{(i,j,k)\in\Omega} \mathcal{L}(Y_{ijk}(X_{ij} - X_{ik}))$$

Convex ERM

Theorem

Suppose

- $\mathcal{L}(\cdot)$ *: 1-Lipschitz*
- $\hat{X}$ *: The optimum of the convex program*

Then, in the above setting,

$$\underbrace{\mathbb{E}_{\Omega,\mathcal{Y}} R(\hat{X})}_{\text{Expected risk of } \hat{X}} \leq \underbrace{\inf_{\{X : \|X\|_* \leq \sqrt{\lambda d_1 d_2}\}} R(X)}_{\text{The best expected risk}} + \underbrace{C\kappa \sqrt{\frac{\lambda(d_1+d_2)}{m}} \log(d_1+d_2)}_{\text{Excess risk bound}}.$$

$O(r \log^2 d)$ comparisons/user are sufficient if $d_1, d_2 \approx d$.

Consistency in the Multi-user BTL model

- Assume there is a ground-truth $X^* \in \mathbb{R}^{d_1 \times d_2}$.

$$\Pr\{Y_{ijk} = +1\} = \frac{e^{X^*_{ij} - X^*_{ik}}}{1 + e^{X^*_{ij} - X^*_{ik}}}$$

- ML estimation : Solving the ERM with $\mathcal{L}(z) = \log(1 + \exp(z)) - z$.

Corollary

Suppose that $\mathcal{Y} \sim BTL(X^*)$ where $\|X^*\|_* \leq \sqrt{\lambda d_1 d_2}$. Under the sampling assumption,

$$\frac{1}{d_1 d_2^2} D(\mathbb{P}_{X^*} \| \mathbb{P}_{\hat{X}}) \leq C\kappa \sqrt{\frac{\lambda(d_1 + d_2)}{m}} \log(d_1 + d_2).$$

Can recover the true X^* with $O(r \log^2 d)$ comparisons/user.

ERM Lower Bound

Is the $O(r \log^2 d)$ sample complexity good?

Theorem

For any estimator $\hat{X}$ as a function of Ω and $\mathcal{Y}$, there exists X^* such that[a]

$$\mathbb{E}_{\Omega,\mathcal{Y}} R(\hat{X}) \geq R(X^*) + c \min\left\{1, \sqrt{\frac{\lambda(d_1 + d_2)}{m}}\right\},$$

with probability at least $\frac{1}{2}$.

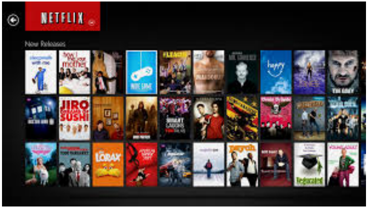
[a] Under the assumption $\mathcal{L}'(0) < 0$, $\lambda \geq 1$, and $m \geq d_1 + d_2$

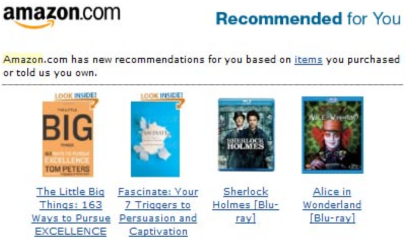
Need at least $O(r)$ comparisons/user.

However, In Practice..

The size of user-item matrices?

- Netflix prize : 480,000 users $\times$ 17,000 movies
- Personalization datasets often even larger



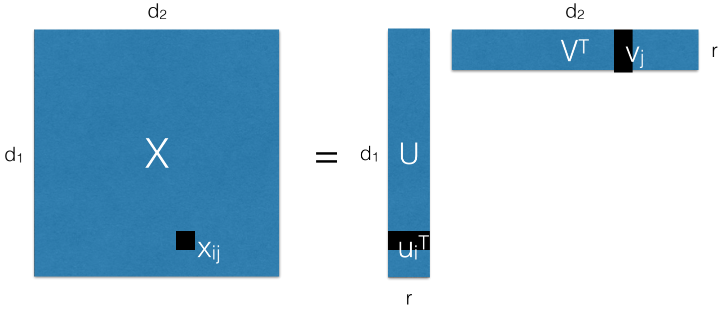


Convex optimization needs to train and store $10^{10} \sim 10^{15}$ parameters.

Non-Convex Algorithm

$$\underset{U \in \mathbb{R}^{d_1 \times r}, V \in \mathbb{R}^{d_2 \times r}}{\text{minimize}} \sum_{(i,j,k) \in \Omega} \mathcal{L}(Y_{ijk} \cdot u_i^\top (v_j - v_k))$$

- Train a factored form $X = UV^\top$ ($X_{ij} = u_i^\top v_j$)

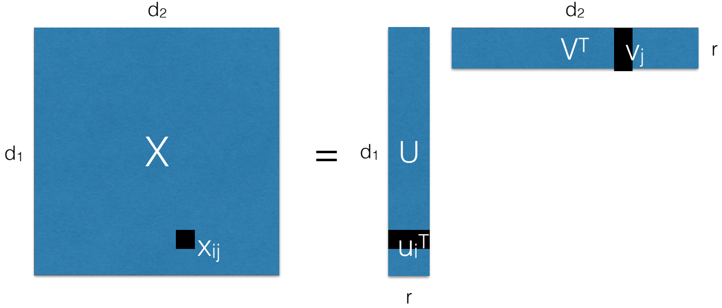


Now only $(d_1 + d_2)r$ parameters

Non-Convex Algorithm

$$\underset{U \in \mathbb{R}^{d_1 \times r}, V \in \mathbb{R}^{d_2 \times r}}{\text{minimize}} \sum_{(i,j,k) \in \Omega} \mathcal{L}(Y_{ijk} \cdot u_i^\top (v_j - v_k)) + \frac{\lambda}{2}(\|U\|_F^2 + \|V\|_F^2)$$

- Add regularizer to control overfitting



Non-Convex Implementation

- Updating U (while V is fixed) : Ranking SVM [Joachims, 2002]

  "Find the personalized weight vector u_i for each user."

$$\forall i, \quad u_i \leftarrow \arg\min_{u \in \mathbb{R}^r} \frac{\lambda}{2}\|u\|_2^2 + \sum_{j,k:(i,j,k)\in\Omega} \mathcal{L}(Y_{ijk} \cdot u^\top (v_j - v_k))$$

Can be decomposed into d_1 independent r-dimensional SVMs

- Updating V (while U is fixed) "Embed d_2 item vectors into $\mathbb{R}^r$."

$$V \leftarrow \arg\min_{V \in \mathbb{R}^{d_2 \times r}} \left\{ \frac{\lambda}{2}\|V\|_F^2 + \sum_{(i,j,k)\in\Omega} \mathcal{L}(Y_{ijk} \cdot \langle A^{(ijk)}, V \rangle) \right\}$$

Also a SVM! but too large ($d_1 \times r$ dimensional)

- Solution: dual coordinate ascent still $O(r)$

Non-Convex Implementation

Dual Coordinate Descent [Hsieh et al., 2007]

- Dual problem

$$\min_{\beta \in \mathbb{R}^{|\Omega|}, \beta \geq 0} \frac{1}{2} \left\| \sum_{(i,j,k) \in \Omega} \beta_{ijk} A^{(ijk)} \right\|_F^2 + \frac{1}{\lambda} \sum_{(i,j,k) \in \Omega} \mathcal{L}^*(-\lambda \beta_{ijk})$$

- Coordinate descent : Fix all but one variables, and optimize.

$$\delta^* \leftarrow \arg\min_{\delta \geq -\beta_{ijk}} \frac{1}{2} \left(\|v_j + \delta Y_{ijk} u_i\|_2^2 + \|v_k - \delta Y_{ijk} u_i\|_2^2 \right) + \mathcal{L}^*(-\lambda(\beta_{ijk} + \delta)),$$

$$\beta \leftarrow \beta + \delta^*,$$

$$v_j \leftarrow v_j + \delta^* Y_{ijk} u_i,$$

$$v_k \leftarrow v_k - \delta^* Y_{ijk} u_i.$$

$O(r)$ computation

Alternating SVM (AltSVM)

While not converged do

1. Stochastic dual coordinate descent for V.
   - For $t = 1, \ldots, T$,
   - Randomly pick $(i, j, k) \in \Omega$.
   - Do coordinate descent for the dual variable β_{ijk}.
   - Update v_j and v_k. $O(r)$ computation
2. Stochastic dual coordinate descent for U.
   - For $t = 1, \ldots, T$,
   - Randomly pick $(i, j, k) \in \Omega$.
   - Do coordinate descent for the dual variable α_{ijk}.
   - Update u_i. $O(r)$ computation

Alternating SVM (AltSVM)

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1. Stochastic dual coordinate descent for V.
   - For $t = 1, \ldots, T$,
   - Randomly pick $(i, j, k) \in \Omega$.
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   - Update v_j and v_k. $O(r)$ computation
2. Stochastic dual coordinate descent for U.
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   - Randomly pick $(i, j, k) \in \Omega$.
   - Do coordinate descent for the dual variable α_{ijk}.
   - Update u_i. $O(r)$ computation

Decomposability does not matter.

Paralellization

Each coordinate descent updates at most $2r$ out of $(d_1 + d_2)r$ variables.

- Can apply parallel asynchronous stochastic DCD without locking.[1]

# cores	1	2	4	8	16
Time(seconds)	963.1	691.8	365.1	188.3	111.0
Speedup	1x	1.4x	2.6x	5.1x	8.7x

Table : Scalability of AltSVM on the binarized MovieLens1m dataset.

[1] Hsieh, Yu, and Dhillon, "PASSCoDe: Parallel Asynchronous Stochastic Dual Coordinate Descent," ICML 2015.

Experiments

We compare our algorithm (with hinge loss) to

- CofiRank [Weimer et al., NIPS'07]
- Local Collaborative Ranking [Lee et al., WWW'14]
- Robust Binary Ranking [Yun et al., NIPS'14]
- SGD : Stochastic Gradient Descent on our non-convex formulation.
- Global ranking : Aggregate all comparisons and provide one ranking.

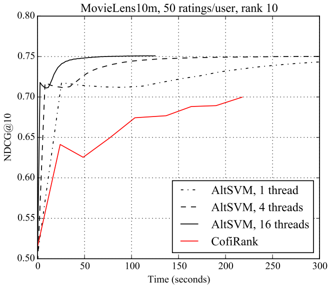
Datasets

- Binarized MovieLens1m : 6,040 $\times$ 3,900 movies, 1m ratings
- MovieLens10m : 71,567 users $\times$ 10,681 movies, 10m ratings
- Netflix prize : 480,000 users $\times$ 17,000 movies, 100m ratings

Experimental results - Rating data

- Compared in terms of NDCG@10
- AltSVM takes all non-tying comparisons from the sampled ratings

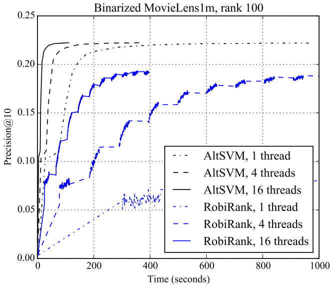
Datasets	# ratings/user	AltSVM	SGD	Global	CofiRank	LCR
MovieLens10m	20	0.7059	0.6977	**0.7264**	0.7076	0.6977
	50	**0.7508**	0.7452	0.7176	0.6977	0.6940
	100	**0.7692**	0.7659	0.7101	0.6754	0.6899
Netflix	20	0.7132	-	**0.7605**	0.6615	-
	50	**0.7642**	-	**0.7640**	0.6527	-
	100	**0.8007**	-	0.7656	0.6385	-



Experimental results - Binary data

- Compared in terms of Precision@K
- AltSVM takes C non-tying comparisons for each user.

Precision@	AltSVM $C = 1000$	AltSVM $C = 2000$	AltSVM $C = 5000$	SGD	RobiRank
1	0.2165	0.2973	**0.3635**	0.1556	0.3009
2	0.1965	0.2657	**0.3297**	0.1498	0.2695
5	0.1572	0.2097	**0.2697**	0.1236	0.2300
10	0.1265	0.1709	**0.2223**	0.1031	0.1922
100	0.0526	0.0678	**0.0819**	0.0441	0.0781



Summary

- Two algorithms for collaborative ranking from pairwise comparisons
- Convex relaxation
  - $O(r \log^2 d)$ sample complexity for arbitrarily small excess risk
- Alternating SVM through Stochastic Dual Coordinate Descent
  - Scalable and outperforming existing algorithms in ranking measures